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Abstracts

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Zsolt's Choices

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Much of the work of Zsolt Tuza deals with choice numbers of graphs. This led to the above choice of the subject of the lecture. I will describe some recent and less recent results and questions about choosability of graphs, focusing on a construction in a joint work with Kostochka, Reiniger, West and Zhu of sparse bipartite graphs of high girth and large choice number.

Short Cycles and Long Cycles through Specified Vertices of a Polyhedral Graph

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We discuss the existence of a cycle through prescribed vertices of a polyhedral graph and present bounds on its possible length.

Transversals in Uniform Linear Hypergraphs

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The transversal number $\tau(H)$ of a hypergraph H is the minimum number of vertices that intersect every edge of H. A k-uniform hypergraph has all edges of size k. For $k \geq 2$, let $\mathcal{H}_k$ denote the class of all k-uniform hypergraphs. Zsolt Tuza [Discrete Math. 86 (1990), 117–126] proposed the problem of determining the best possible constants c_k (which depends only on k) such that $\tau(H) \leq c_k(n_H + m_H)$ for all $H \in \mathcal{H}_k$, where $n_H = |V(H)|$ and size $m_H = |E(H)|$. It is known that $c_2 = \frac{1}{3}$, $c_3 = \frac{1}{4}$, $c_4 = \frac{2}{9}$, while the precise value of c_k is not yet known for any $k \geq 5$. Alon determined the asymptotic behaviour of c_k as k grows, and showed that $c_k = (1 + \mathrm{o}(1))\,(\ln(k)/k)$ as $k \to \infty$.

In this talk, we discuss an analogous problem for linear hypergraphs where every two distinct edges intersect in at most one vertex. Let $\mathcal{L}_k$ denote the class of k-uniform linear hypergraphs. We consider the problem of determining the best possible constants q_k (which depends only on k) such that $\tau(H) \leq q_k(n_H + m_H)$ for all $H \in \mathcal{L}_k$. It is known that $q_2 = \frac{1}{3}$ and $q_3 = \frac{1}{4}$. We show that $q_4 = \frac{1}{5}$, implying that the bound on the transversal number for linear hypergraphs is better than for non-linear hypergraphs in the case of 4-uniformity. Using the affine plane $AG(2,4)$ of order 4, we show there are a large number of densities of hypergraphs $H \in \mathcal{L}_4$ such that $\tau(H) = \frac{1}{5}(n_H + m_H)$. Key to our proof is the new technique of the deficiency of a hypergraph we introduce. We show that the asymptotic behaviour of q_k as k grows is the same as that of c_k, namely of the order $\ln(k)/k$.

1. M. A. Henning and A. Yeo, Transversals in uniform linear hypergraphs, manuscript. http://arxiv.org/abs/1802.01825.

The domination number of the graph defined by two levels of the n-cube

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Consider all k-element subsets and ℓ-element subsets ($k > \ell$) of an n-element set as vertices of a bipartite graph. Two vertices are adjacent if the corresponding ℓ-element set is a subset of the corresponding k-element set. Let $G_{k,\ell}$ denote this graph. The domination number of $G_{k,1}$ will exactly be determined. We also give lower and upper estimates on the domination number $G_{k,2}$ and pose a conjecture of asymptotic nature.

Properties of DP-coloring of graphs and multigraphs

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A DP-k-coloring of a (multi)graph G is not a coloring, but an independent set in an auxiliary graph $H(G, k)$. Nevertheless, DP-coloring is a generalization of list coloring and possesses many properties of it. It was introduced by Dvořák and Postle to resolve a question of Borodin on list-coloring of planar graphs.

We discuss several features of DP-coloring of graphs and multigraphs and compare them with those of list coloring. We also discuss some open questions on DP-coloring.

This is joint work with A. Bernshteyn, S. Pron and X. Zhu.

Subgraphs with Specified Color Properties in Vertex- or Edge-colored Graphs

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Here we deal with problems in edge- or vertex colored graphs. As an example, the Web graph may be considered as a vertex-colored graph where the color of a vertex represents the content of the corresponding page (red for mathematics, yellow for physics, etc.). When the edges/vertices of graphs are colored, then we talk about c-edge/vertex colored graphs (c is the number of colors), models which in fact generalize various classes of graphs. In general, one can observe that problems related to colored graphs often consist in finding subgraphs such as paths, cycles and trees, with, in addition, specified constraints on colors.

In the case of c-edge-colored graphs, one such natural constraint is the proper coloring one, i.e., any two adjacent edges differ on colors. Given such an edge-colored graph, original problems correspond to extracting subgraphs, for example, Hamiltonian and Eulerian paths or cycles, trees, etc., colored in this specified pattern. Although a large body of work has already been done, in the most of these researches the number of colors is restricted to two. For instance, while it is well known how to find efficiency a properly edge colored Hamiltonian cycle in a 2-edge colored complete graph, it is a long-standing question how to find such cycles in complete graphs whose edges are colored by any number of colors. Here we survey a series of results concerning proper subgraphs in c-colored graphs, for arbitrarily c.

In the case of vertex-colored graphs, we deal with tropical subgraphs, a concept with direct applications to the web graph and in bioinformatics. More precisely, given a vertex-colored graph, a tropical subgraph (induced or otherwise) is defined to be a subgraph where each color of the initial graph appears at least once. Notice that in a tropical subgraph, adjacent vertices can receive the same color, thus a tropical subgraph may not be properly colored. Potentially, many graph invariants, such as the domination number, the vertex cover number, maximum matchings, independent sets, connected components, shortest paths, etc. can be studied in their tropical version. This notion is close to, but somewhat different from the colorful concept (all colors of the subgraph are different) used for paths and elsewhere in vertex-colored graphs. It is also related to the concepts of color patterns (the subgraph has fixed color patterns) used in bio-informatics. Here we explain some results on our ongoing work on tropical dominating sets, vertex covers, connected subgraphs, maximum matchings and tropical homomorphisms.

Discrete optimization in transportation and logistics

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Information and communication technologies (ICT) nowadays enable users to access, store, transmit, and manipulate information. ICT have created new opportunities to the economic system and to the society. A systemic approach to problems and advanced mathematical methods are even more vital than in the past. In this talk emerging research directions in the field of transportation and logistics will be discussed, in particular directions in which discrete optimization and mixed integer linear programming models play a key role towards solutions that are more integrated or coordinated. Integrated problems in supply chain management and logistics will be discussed that are aimed at optimizing the system behavior. Location routing problems, loading and routing problems, multi-echelon routing problems, inventory routing problems can reduce the costs substantially with respect to the sequential solution of the sub-problems. On the other hand, collaboration has been enhanced by advances in ICT that have enabled information sharing, for example among carriers. In a centralized collaboration scheme, a central decision maker can redistribute customers and/or logistic assets among carriers. This decision maker may be a third party who acts in a non-partisan way or may be a large carrier that resorts to other carriers to manage all its orders and customers. Collaboration schemes will be discussed together with their benefits. Some research directions will be discussed also in the field of passenger transportation.

List colorings of graphs

Margit Voigt
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Let $G = (V, E)$ be a graph, and let $k \geq 0$ be an integer. A *list-assignment* L of G is a function that assigns to each vertex v of G a set (list) $L(v)$ of colors. We say that L is a k-*assignment* if $|L(v)| = k$ for all $v \in V$. The graph G is called *(a,b)-list colorable* if for every a-assignment we can choose color sets $C(v) \subseteq L(v)$ such that $|C(v)| = b \ \forall v \in V$ and $C(v) \cap C(w) = \emptyset \ \forall vw \in E$.

The following questions were posed by Erdős, Rubin and Taylor in 1979. If G is (a, b)-list colorable, does it follow that G is (am, bm)-list colorable? If G is (a, b)-list colorable, and $\frac{c}{d} > \frac{a}{b}$, does it follow that G is (c, d)-list colorable? We will discuss old and new results concerning these questions.

In the second part of the talk we consider a special kind of list assignments recently introduced by Choi and Kwon. A *t-common k-assignment* is a k-assignment satisfying $|\bigcap_{v \in V(G)} L(v) \geq t|$. We investigate planar graphs obtaining a strengthening of the result on non-4-list colorable planar graphs.

32 years of walk, searching for the optimum

Gábor Bacsó
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Since 1986, we were investigating a lot of problems arising from combinatorial optimization with Zsolt together.

Most of the concepts here will be defined in the talk only.

Wolk and Seinsche characterized some simple graph classes by domination properties. Continuing their work, we gave a similar characterization of the P_5- and C_5-free graphs. This characterization is short. The equivalent property is — to have some dominating clique in every connected induced subgraph. Moreover, a weaker property was found for the case when P_5 is forbidden only.

In 2010, Ingo Schiermeyer and Bert Randerath found a subexponential algorithm for the stable set problem in P_5-free graphs. This was a breakthrough. And they applied our innocent result on P_5-free graphs.

Roughly in 1988, we have put the following much more general question. Given a class $\mathcal{D}$ of connected graphs, determine the graphs, hereditarily dominated by $\mathcal{D}$. About 18 years after, we both have found the complete description — independently, using different methods.

$\mathcal{C}$-coloring is one of the frequently investigated non-traditional colorings of hypergraphs. The *upper chromatic number* is the maximum number of possible colors in such a coloring. This number is connected with the well-known 2-covering number. For the case of finite projective planes, we found thorough connection between the two. Moreover, this was possible not only for coordinatizable planes, but for each of them. Later, with Tamás Héger and Tamás Szőnyi, we obtained that, for coordinatizable planes equality holds, for almost every choice of the parameters.

$\mathcal{C}$-coloring was defined in the frame of *mixed* hypergraphs, introduced by V. Voloshin. The so called *bi-hypergraphs* yield an important subproblem. We characterized the uniquely colorable ones among them.

If I have time, probably some non-traditional colorings of graphs and the clique-coloring of line graphs will also be mentioned.

Modern bin packing: overview of the results of 40 years research in Szeged

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Bin packing is one of the most deeply studied problem in the field of combinatorial optimization. The one-dimensional version can be defined as follows. Let $L = \{a_1, a_2, \ldots, a_n\}$ be a list of n items, with sizes $s(a_i) \in (0, 1]$, $i = 1, \ldots, n$. The task is to assign the items to the *minimal* number of unit capacity bins, subject to the constraint that the total size of the items assigned to any bin is *at most* 1. The bin packing problem has been extensively studied in the last 40 years. Since the problem has various application possibilities a large number of variants exist. Many important research results have been obtained. This problem has played an important role in the reasearch work of the combinatorial optimization groups at the University of Szeged. In this talk we overview the most important results connecting to this work. Specifically, we deal with the online verson of the problem and approximate algorithms.

Greedy extensions of graph classes and their applications

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Ewa Drgas-Burchardt
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In this talk we consider a variant of graph partitioning problem consisting in partitioning the vertex set into the minimum number of sets such that each of them induces a graph in a fixed hereditary class of graphs. We will discuss the computational complexity of several problems arising when such partitions are generated by the greedy algorithm. Some of these problems are computationally hard, while for others, we present new polynomial-time algorithms. We also show how greedy partitioning can be used to step towards a general technique of designing extensions of graph classes along with new polynomial-time approximation algorithms that solve various minimization problems for inputs in new classes (by designing an extension of a given class we mean determining the set of minimal forbidden graphs that characterize our new class). We also present new results on the so-called fine-grain extensions as well as their applications in obtaining new online χ-bounded classes. Finally we remark on applications in scheduling.

Grundy domination in graphs

Boštjan Brešar
University of Maribor, Slovenia

Consider a sequence $S = (v_1, \ldots, v_k)$ of vertices of a graph G that is obtained while constructing a dominating set of G; in particular, every vertex not in S is adjacent to a vertex in S. A natural condition is that for all i, $i \geq 2$, we have $N[v_i] \setminus \bigcup_{j=1}^{i-1} N[v_j] \neq \emptyset$, and if S obeys this condition we call it a *dominating sequence* in G. While the minimum length of such a sequence in G clearly coincides with the domination number, $\gamma(G)$, we call the maximum length of a dominating sequence in G the *Grundy domination number* of G, and denote it by $\gamma_{gr}(G)$.

The concept and some of its variations have been studied in a number of recent papers. In this talk, we give a brief overview on the state-of-the-art of Grundy domination invariants, and present their connections with other established concepts such as zero forcing sets and the minimum rank of a graph.

Equitable list vertex arboricity of d-dimentional grids

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Janusz Dybizbański, Hanna Furmańczyk
University of Gdańsk, Poland

A graph G is equitably k-list arborable if for any k-uniform list assignment L, there is an equitable L-colouring of G whose each colour class induces an acyclic graph. The smallest number k for which G is equitably k-list arborable is named the equitable list vertex arboricity of G and is denoted by $\rho_l^=(G)$. In 2016 Zhang posed the conjecture that if $k \geq \lceil(\Delta(G)+1)/2\rceil$ then G is equitably k-list arborable. We present some new tools that are helpful in determining values of k for which a general graph is equitably k-list arborable. We use them to prove Zhang's conjecture for d-dimensional grids where $d \in \{2, 3, 4\}$ and give new bounds on $\rho_l^=(G)$ for general graphs and for d-dimensional grids with $d \geq 5$.

Independent 3-domination in 2-trees

Mieczysław Borowiecki and Anna Fiedorowicz
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Let G be a given graph, $S \subseteq V(G)$. For an integer $k \geq 1$, the set S is called *k-dominating* if every vertex not in S has at least k neighbors in S. S is *independent*, if no two vertices of S are adjacent. If we require S to be both dominating and independent, we get the notion of *independent domination.* Thus, S is called an *independent k-dominating set* if is both k-dominating and independent.

The problem of independent k-dominating set and its generalizations get a lot of attention. In particular, in 2005 Blidia, Chellali and Favaron studied it in the class of trees and they characterized all trees having independent 2-dominating set.

We focus on independent 3-domination in the class of 2-trees and present a construction of all 2-trees for which there exists an independent 3-dominating set.

Kneser ranks of random graphs and minimum difference representations

Zoltán Füredi
Alfréd Rényi Institute of Mathematics, Budapest, Hungary

After a short overview of graph representations (a very important topic of graph theory) we consider the following problem.

Every graph $G = (V, E)$ is an induced subgraph of some Kneser graph of rank k, i.e., there is an assignment of (distinct) k-sets $v \mapsto A_v$ to the vertices $v \in V$ such that A_u and A_v are disjoint if and only if $uv \in E$. The smallest such k is called the *Kneser rank* of G and denoted by $f_{\text{Kneser}}(G)$. As an application of a result of Frieze and Reed concerning the clique cover number of random graphs we show that for constant $0 < p < 1$ there exist constants $c_i = c_i(p) > 0$, $i = 1, 2$ such that $G \in \mathcal{G}(n, p)$ satisfies with high probability

$$c_1 n/(\log n) < f_{\text{Kneser}}(G) < c_2 n/(\log n).$$

We apply this for other graph representations defined by Boros, Gurvich and Meshulam.

A *k-min-difference representation* of a graph G is an assignment of a set A_i to each vertex $i \in V(G)$ such that

$$ij \in E(G) \;\Leftrightarrow\; \min\{|A_i \setminus A_j|, |A_j \setminus A_i|\} \geq k.$$

The smallest k such that there exists a k-min-difference representation of G is denoted by $f_{\min}(G)$. Balogh and Prince proved in 2009 that for every k there is a graph G with $f_{\min}(G) \geq k$. We prove that there are constants $c_1'', c_2'' > 0$ such that $c_1'' n/(\log n) < f_{\min}(G) < c_2'' n/(\log n)$ holds for almost all bipartite graphs G on $n + n$ vertices.

Joint work with Ida Kantor.

Triangles and cliques in graphs

Ervin Győri
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In this lecture, we study old and new problems concerning triangles and cliques. The maximum number of edge disjoint ones are studied as well as covering the edges of a graph by cliques.

Recently, Keszegh and the speaker proved the quarter of a century old conjecture of Erdős that every K_4-free graph with n vertices and $t_2(n) + k$ edges contains k pairwise edge disjoint triangles. The half a century old history of the problem starting with another similar question of Erdős and a 30 year old problem of Tuza and the speaker will be summarized too.

Other recent papers are related to coverings by minimum number of triangles or cliques (of given type). The local clique cover number of G, denoted by $lcc(G)$, is defined as the smallest integer k, for which there exists a clique covering for $E(G)$ such that the number cliques containing v is at most k, for every vertex $v \in V(G)$. E.g., with Bujtás, Davoodi, and Tuza, we proved that $lcc(G) + \chi(G) \leq n + 1$ if G is a claw-free graph with n vertices. These results will be presented in more details in the lecture of Davoodi.

We also present (shortly) some theorems (by Bujtás, Davoodi, Ding, Tuza, Yang, and the speaker) on triangle coverings of graphs.

Facial $L(2,1)$-edge-labelings of trees

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Let G be a plane graph. A facial path of G is any path which is a consecutive part of the boundary walk of a face of G. Two edges e_1 and e_2 of G are facially adjacent if they are consecutive on a facial path of G. Two edges e_1 and e_3 are facially semi-adjacent if they are not facially adjacent and there is a third edge e_2 which is facially adjacent with both e_1 and e_3, and the edges e_1, e_2, e_3 are consecutive (in this order) on a facial path. An edge-labeling of G with labels $1, 2, \ldots, k$ is a facial $L(2,1)$-edge-labeling if facially adjacent edges have labels which differ by at least 2 and facially semi-adjacent edges have labels which differ by at least 1. The minimum k for which a plane graph admits a facial $L(2,1)$-edge-labeling is called the facial $L(2,1)$-edge-labeling index.

In this talk, we show that the facial $L(2,1)$-edge-labeling index of any tree T is at most 7; moreover, this bound is tight. In the case when T has no vertex of degree 3 the upper bound for this parameter is 6, which is also tight. If T is without vertices of degree 2 and 3, then its facial $L(2,1)$-edge-labeling index is at most 5; moreover, this bound is also tight. Finally, we characterize all trees having facial $L(2,1)$-edge-labeling index exactly 4.

Domination game and minimal edge cuts

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The domination game is played on a graph G by Dominator and Staller. They take turns choosing a vertex from G such that at least one previously undominated vertex becomes dominated. The game is over when G becomes dominated. Dominator wants to minimize the number of vertices played, and Staller wishes to maximize it. If Dominator starts the game and both players play optimally, then the number of moves played is the *game domination number* $\gamma_g(G)$ of G. In this talk a relationship between the domination game and minimal edge cuts will be presented. In particular, if C a minimum edge cut of a connected graph G, then $\gamma_g(G) \leq \gamma_g(G \setminus C) + 2\kappa'(G)$. Double-Staller graphs will be introduced in order to show that this upper bound can be attained for graphs with a bridge. The family of known traceable graphs whose game domination numbers are at most one-half their order will be extended.

Minimal unavoidable sets of cycles in plane graphs

Tomáš Madaras and Martina Tamášová
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A set $\mathcal{S}$ of graphs is minimal unavoidable in an infinite graph family $\mathcal{G}$ if each graph $G \in \mathcal{G}$ contains a graph from $\mathcal{S}$ and, for each proper subset $\mathcal{S}' \subset \mathcal{S}$, there exists an infinite subfamily $\mathcal{G}' \subset \mathcal{G}$ such that no graph from $\mathcal{G}'$ contains a graph from $\mathcal{S}'$. We present an overview of previously known and recent results on minimal unavoidable sets of cycles in plane graphs of minimum degree at least 3 as well as in plane graphs with higher minimum degree or prescribed minimum edge weight.

Gröbner Bases and Combinatorics, some examples

Lajos Rónyai
MTA SzTAKI, BME, Hungary

Gröbner bases and related objects (such as leading monomials, normal sets, Hilbert functions) are important tools in the theoretical and computational study of ideals in multivariate polynomial rings. Algebraic techniques based on these notions can be applied also to ideals which originate from combinatorial structures. For example, one can consider the ideal of polynomial functions vanishing on the characteristic vectors of a (full) uniform set family, and study the Gröbner objects of this ideal. The talk will present results and applications of this type.

Some Ramsey Type Results

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We will present some old and recent Ramsey Type results, mostly proved using the so-called 'monochromatic connected matching' method invented by Luczak. This is a very powerful method, based on the Szemerédi Regularity Lemma. We will present the method itself, and show several examples when it occurred to be in particularly useful.

Better Algorithms for Online Bin Stretching

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Online Bin Stretching is a semi-online variant of bin packing in which the algorithm has to use the same number of bins as an optimal packing, but it is allowed to slightly overpack the bins. The goal is to minimize the amount of overpacking, i.e., the maximum size packed into any bin.

We survey the recent progress in this area. As a main result, we sketch an algorithm for Online Bin Stretching with a stretching factor of 1.5 for any number of bins. We build on previous algorithms and use a two-phase approach. Our analysis combines amortization over the bins with the help of two weight functions. We also present an algorithm with a stretching factor of 11/8=1.375 and a lower bound of 45/33=1.363 for three bins. This and additional lower bounds were obtained using computer search.

Joint work with Martin Böhm, Rob van Stee, and Pavel Veselý.

Batch Scheduling of Nonidentical Job Sizes with Minsum Criteria

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We concerns the problem of scheduling jobs with unit processing time and nonidentical sizes on single or parallel identical batch machines. The objective is to minimize the total completion time of all jobs. We show that the worst-case ratio of the algorithm based on the bin-packing algorithm First Fit Increasing (FFI) is at most $\frac{3}{2}$ for the single machine case, and is no more than $\frac{7}{4}$ for the parallel machines case. We also present two heuristics and compare the performances of the algorithms by numerical experiments.

On the Erdős Multiplication Table Problem

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Let

$$P_m(n) := \left\{ \prod_{i=1}^{n} i^{\alpha_i} \,\middle|\, \alpha_i \in \mathbb{N}_0 \text{ and } \sum_{i=1}^{n} \alpha_i = m \right\}$$

be the set of products of m numbers from the set $\{1, \ldots, n\}$. In 1955 Erdős posed the problem of determining the order of magnitude of $|P_2(n)|$. This so-called Erdős Multiplication Table Problem was settled in 2008 by Ford. Koukoulopoulos determined the order of magnitude of $|P_m(n)|$. Recently, Darda and Hujdurović asked if, for fixed n, $|P_m(n)|$ is a polynomial in m of degree $\pi(n)$ - the number of primes not larger than n. Motivated by this question we present and discuss a connection between Ehrhart Theory, Commutative Algebra and the Erdős Multiplication Table Problem. We prove the conjecture of Darda and Hujdurović for each fixed n if m is large.

Joint work with Robert Scheidweiler.

Mathematical problems in designing a post-disaster relief system

Béla Vizvári
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The most serious disaster of a metropolitan city is earthquake which is in the focus of this study. For example, San Francisco, Los Angeles, Istanbul, Tehran, and Tokyo may face serious earthquake. Lisbon was seriously damaged by an earthquake and its tsunami in 1755. A relief system which is prepared to the post-disaster period can save lives and reduce suffering. In this talk a relief system is discussed which uses Unmanned Aerial Vehicle (UAV) technology. UAVs can be used for relief distribution, reconnaissance, patrolling, and measuring. The relief items are transported to distribution points covering the city. The transportation consists of waves as the demand changes in time. The system is controlled by the Disaster Command Center (DCC). DCC controls relief distribution, emergency vehicles, and local police. DCC has a multi-technology, and multi-channel communication system. It communicates to the local population, and the units of the relief system. DCC up-dates a data-basis in real time mode. Decisions and information provided to the local population reflect always the latest data. The talk concentrates on new problems needing mathematical models and methods to be solved. The pre-assignment of personnel to operation rooms of hospitals, real-time minimal path finding in a dynamically changing environment, scheduling emergency vehicles to transport injured peoples to hospitals, etc. belong to these problems.

Nonrepetitive colorings of the plane

Joanna Chybowska-Sokół, Michał Dębski,
Konstanty Junosza-Szaniawski, Barbara Pilat,
Urszula Pastwa, Krzysztof Węsek
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A *repetition* in a sequence is a pair of two identical adjacent blocks. For example, *abab* is a repetition. A classic theorem by Axel Thue states that using just 3 symbols one can construct an infinite sequence without a repetition. In this work, we consider Thue-type questions concerning coloring of the Euclidean plane, related to the famous Hadwiger-Nelson problem. Fix $b \geq 1$. Let a sequence of points on a line in $\mathbb{R}^2$ with consecutive distances from the interval $[1, b]$ be called a *line b-path*. How many colors are needed for a coloring of $\mathbb{R}^2$ such that the sequence of colors on every line b-path is nonrepetitive? In our talk, we will show how to approach such questions.

As a key tool, we use a variation of the classic Thue considerations called grasshopper avoidance of repetitions. Roughly speaking, the aim is to build a sequence that avoids repetition occurrence not only as a block but also as a subsequence with a limited amount of 'jumps'. This tool, however, raises questions interesting on their own as well.

Crumby Colorings

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When the first proof of the 4-color Theorem was announced in the 1970's, Wegner initiated the study of the square chromatic number of planar graphs. We recall the case $\Delta \leq 3$: For any subcubic planar graph G, the square of G is 7-colorable.

Recently, Thomassen published his proof of Wegner's conjecture. He formulated the following attractive conjecture, which would imply Wegner's: Every 3-connected cubic graph has a red-blue vertex coloring such that the blue subgraph has maximum degree 1 (that is, it consists of a matching and some isolated vertices) and the red subgraph has minimum degree at least 1 and contains no 3-edge path.

Since these monochromatic parts are all very small, we use the term *crumby coloring* for short. In Wegner's conjecture the Petersen graph is extremal. Therefore, we first prove the existence of a crumby coloring for generalised Petersen graphs.

We conjecture that every subcubic graph has a crumby coloring. We confirm this statement for all subcubic trees.

If time permits, we mention various other problems in the same fashion.

Cross-intersecting families

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A typical problem in extremal set theory is to determine how small or how large a parameter of a system of sets can be. The Erdős–Ko–Rado Theorem is a classical result in this field. A variant of the Erdős–Ko–Rado problem is to determine the maximum sum or the maximum product of sizes of k *cross-t-intersecting* subfamilies $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_k$ of a given family $\mathcal{F}$ of sets (i.e. for $i \neq j$, each set in $\mathcal{A}_i$ intersects each set in $\mathcal{A}_j$ in at least t elements). This natural problem has recently attracted much attention. Solutions have been obtained for various important families, such as power sets, levels of power sets, levels of hereditary families, families of integer sequences, families of permutations, and families of vector spaces. The talk will provide an outline of these results, focusing in particular on recent results for levels of hereditary families.

Spanning trees and (logarithmic) least squares optimality

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Given a simple undirected graph and an associated incomplete skew-symmetric matrix $\mathbf{B}$, the goal is to assign values (u_i) to the nodes such that the sum of the functions $(b_{ij} - u_i + u_j)^2$ is minimized. We show that the solution is unique if and only if the graph is connected and the optimal solution is the arithmetic mean of solutions to the same problem but written for each spanning tree of the graph. The problem can be realized by an electric circuit, where every edge includes a resistor of identical resistance, and a voltage source, represented by the corresponding element of matrix $\mathbf{B}$. The calculation of potentials (u_i) with Kirchhoff's rules is shown to be equivalent to the arithmetic mean of potentials calculated from all spanning trees of the electric circuit. Another application is preference modelling in decision theory, where the logarithmic least squares problem is considered for the incomplete pairwise comparison matrix $\mathbf{A} = \exp \mathbf{B}$ (element-wise).

On the fractional version of the domination game

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Given a graph G, a function $f : V(G) \mapsto [0,1]$ is a fractional dominating function if $\sum_{u \in N[v]} f(u) \geq 1$ holds for every vertex v in G. The aim is to minimize the sum $\sum_{v \in V(G)} f(v)$.

A different approach to graph domination is the domination game. It is played on a graph G by two players, namely Dominator and Staller, who take turns choosing a vertex such that at least one previously undominated vertex becomes dominated. The game is over when all vertices are dominated. Dominator wants to finish the game as soon as possible, while Staller wants to delay the end. Assuming that both players play optimally and Dominator starts, the length of the game on G is uniquely determined and called the game domination number of G.

In the fractional version of the domination game the steps are ruled by the condition of fractional domination. We prove some basic results on this topic.

Local Clique Covering of Graphs

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Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. An *edge clique covering* for G, denoted by $\mathcal{C}$, is a collection of cliques of G such that every edge of G appears in at least one clique in $\mathcal{C}$. The *valency* of a vertex $v \in V(G)$ with respect to edge clique covering $\mathcal{C}$, denoted by $val_{\mathcal{C}}(v)$, is defined to be the number of cliques in $\mathcal{C}$ containing the vertex v.

We are interested in an edge clique covering of G in which maximum valency of the vertices is minimized, where the minimum is taken over all edge clique coverings of G. This minimum is called the *local clique cover number* of G, which is denoted by $\mathrm{lcc}(G)$. In the other words, we may have

$$\mathrm{lcc}(G) = \min_{\mathcal{C}} \max_{v \in V(G)} val_{\mathcal{C}}(v).$$

The local clique cover number may be interpreted as a variety of different invariants of the graph. For example, $\mathrm{lcc}(G)$ is the minimum integer k for which G is the line graph of a k-uniform hypergraph. R. Javadi conjectured that for every $n-$vertex graph G the two following inequalities hold.

1. $\mathrm{lcc}(G) + \mathrm{lcc}(\overline{G}) \leq n$
2. $\mathrm{lcc}(G) + \chi(G) \leq n + 1$,

where $\overline{G}$ is the complement of G and $\chi(G)$ denotes the chromatic number of G. Among some other results, we show that the first inequality holds if independence number of G is at most two, and the second inequality is true if G is a claw-free graph.

Cubic bi-Cayley graphs over solvable groups are 3-edge-colorable

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Bi-Cayley graphs are graphs admitting a semiregular group of automorphisms with two orbits. A notable class of cubic bi-Cayley graphs is the so-called generalized Petersen graphs. Castagna and Prins proved in 1972 that all generalized Petersen graphs except for the Petersen graph itself can be properly 3-edge-colored. In this talk, we are going to discuss the extension of this result to all connected cubic bi-Cayley graphs over solvable groups. Our theorem is a bi-Cayley analogue of similar results obtained by Alspach, Liu, Zhang and independently by Nedela and Åăkoviera for Cayley graphs and by PotoÄŊnik for vertex-transitive graphs.

Joint work with Roman Nedela.

List Colorings and Graph Polynomials

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Graph polynomial P_G of a given graph G is a multivariable polynomial defined so that points at which it is not vanishing are precisely proper colorings of G. By studying algebraic properties of P_G one may derive useful information on graph colorings. For instance, the celebrated Combinatorial Nullstellensatz of Alon gives a sufficient condition for existence of a coloring from arbitrary lists of appropriate size. I will present some problems and results showing that this approach to graph coloring is perhaps not fully explored.

Joint work with Bartłomiej Bosek, Grzegorz Gutowski, and Oriol Serra.

Effect of predomination and vertex removal on the game total domination number of a graph

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The total domination game was introduced by Henning et al. in 2015 as a game played on a graph G by two players, Dominator and Staller, who alternate taking turns for as long as possible. On each turn one chooses such a vertex in G that totally dominates at least one not yet totally dominated vertex. Dominator tries to minimize and Staller tries to maximize the number of moves. The total number of selected vertices is called the game total domination number, $\gamma_{tg}(G)$.

In this talk we discuss the effect of vertex predomination on the game total domination number. We prove that $\gamma_{tg}(G|v) \geq \gamma_{tg}(G) - 2$ holds for all vertices v of a graph G and present infinite families attaining the equality. To achieve this, some new variations of the total domination game are introduced. The effect of vertex removal is also discussed.

Approximation Schemes for Minimizing the Maximum Lateness on a Single Machine with Release Times under Non-Availability or Deadline Constraints

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We consider four single-machine scheduling problems with release times, with the aim of minimizing the maximum lateness. In the first problem we have a common deadline for all the jobs. The second problem looks for the Pareto frontier with respect to the two objective functions maximum lateness and makespan. The third problem is associated with a non-availability constraint. In the fourth one, the non-availibility interval is related to the operator who is organizing the execution of jobs on the machine (no job can start, and neither can complete during the operator non-availability period). For each of the four problems, we establish the existence of a polynomial time approximation scheme (PTAS).

An Integer Programming Approach to Characterize Digital Disks

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The integer hull of a polyhedral set is the convex hull of the integer points of the set. In most of the cases the integer hull is a polyhedral set. The integer hull can be determined in an iterative way by Chvátal cuts.

Weighted (or chamfer) distances are popular digital distances used in various grids. Triangular chamfer distances based on three weights are defined. A digital disk (or a chamfer ball) of a grid is the set of the elements which are not on a longer distance from the origin than a given finite bound, radius. These disks are well known and well characterized on the square grid, and recently they become a topic of a current research on the triangular grid. The shapes of the disks in the latter case have a great variability.

The inequalities satisfied by the elements of a disk are analyzed if their Chvátal rank is 1. Individual bounds are described completely. It also gives the complete description of some disks. Further inequalities having Chvátal rank 1 are also discussed.

On the complexity of the canonical partition of graphs

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In matching theory it is a basic problem to determine all the edges in a given graph which can be extended to a maximum matching. Such edges are called maximally matchable or allowed edges.The components induced by these allowed edges determine the so-called canonical partition for graphs having a perfect matching. Apart from the graph theory community, researchers in constraint programming have also investigated this problem. The motivation for studying the question from constraint programming point of view is originating from certain constraint propagation methods. In this talk first we will review the methods from the literature for determining the maximally-matchable edges. Then a decomposition theory based on the matching structure is presented with showing the related algorithmic concept for identifying the allowed edges. It will be also shown that the complexity of the new method can be expressed with the help of a graph parameter arising from the developed decomposition structure.

Computational complexity of problem of 3-coloring in the class of claw-free graphs

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Given an integer k, a *k-coloring* of a graph G is a mapping $f : V(G) \to \{1, \ldots, k\}$ such that any two adjacent vertices $u, v \in V(G)$ satisfy $f(u) \neq f(v)$. Vertex coloring is the problem of determining the chromatic number of a graph; it is a well-known NP-hard problem. In fact, even determining if a graph is 3-colorable is NP-complete and it remains NP-complete also in the class of claw-free graphs in general. In the talk we will focus on the computational complexity of the 3-coloring problem in the subclasses of claw-free graphs defined by forbidding additional subgraphs. We extended the results of Kamiński and Lozin, and proved the polynomial-time solvability of the 3-coloring problem in the class of (claw, Φ_2)-free and (claw, Φ_4)-free graphs.

Approximation algorithms for minimum cost globally rigid subgraphs

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Consider a set of nodes on the plane in general position, and a cost function on the vertex pairs, representsing the cost of fixing the distance between these two nodes (e. g. placing a bar between them). The investigated problem is the following: we want to fix some distances between the nodes, so that any two nodes (even the non-fixed ones) remain the same distance from each other, as we have them now. Determine the minimal cost, with which we can reach this. Also, if we have some distances already fixed, what is the minimal number from the remaining, that must be fixed to reach the same result?

Both questions turned out to be **NP**-hard, in the first case even if the cost function is metric. We developed approximation algorithms: the metric cost function, and the minimum size version on the plane can be solved within constant factor to the optimal. Our result relies on combinatorial structural results of the redundantly rigid and globally rigid graphs.

On semitotal domination in graphs

Esther Galby, Andrea Munaro and Bernard Ries
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A set of vertices S of a graph G with no isolated vertex is a *semitotal dominating set* of G if S is a dominating set of G and every vertex in S is within distance 2 of another vertex of S. The *semitotal domination number* of G is the minimum size of a semitotal dominating set of G. This parameter is squeezed between the domination number and the total domination number and was introduced by Goddard, Henning and McPillan.

In the talk, we provide several complexity results on Semitotal Domination (the problem of deciding the semitotal domination number) and on the inapproximability of the optimization version, answering some questions recently formulated by Henning and Pandey.

New combinatorial interpretations of some variants of Stirling and Bell numbers

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Stirling numbers of the first (second) kind count the number of permutations (partitions) of a finite set with a fixed number number of disjoint cycles (blocks), while Bell numbers give the total number of partitions of a finite set.

Several variants of Stirling numbers are known. For instance, the r-Stirling numbers introduced by Carlitz, Broder and Merris, the Whitney numbers by Dowling, or their common generalization, the r-Whitney numbers by Mező. The corresponding Bell-like numbers are called r-Bell, Dowling and r-Dowling numbers, respectively.

In our talk, we present some of our recent results on new combinatorial interpretations of the above mentioned numbers.

This is a joint work with Eszter Gyimesi.

Overview of our combinatorial and graph theoretical results on r-Lah numbers and r-Lah polynomials

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Based on the idea of Carlitz, Broder and Merris, we define a generalization of Lah numbers. The r-Lah number $\left\lfloor {n \atop k} \right\rfloor_r$ counts the number of partitions of an $(n+r)$-element set into $k+r$ ordered subsets such that r distinguished elements belong to distinct ordered blocks. We also define r-Lah polynomials $L_{n,r}(x) = \sum_{k=0}^{n} \left\lfloor {n \atop k} \right\rfloor_r x^k$.

In our talk we give a detailed overview of the properties of r-Lah numbers and r-Lah polynomials. We derive several combinatorial identities, for example, recurrences, explicit formula, exponential generating function, log-concavity, etc. It turns out that the roots of r-Lah polynomials are negative real numbers. We derive an upper bound for the absolute value of the smallest root, we also compute the real magnitude of this root via computational methods, and study its asymptotic behaviour.

Finally we discuss the graph theoretical interpretations of r-Lah numbers and r-Lah polynomials.

This is a joint work with Gábor Nyul.

Multi-Symbol Forbidden Configurations

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An r-matrix is a matrix with symbols in $\{0, 1, \ldots, r-1\}$. A matrix is simple if it has no repeated columns. For a family of r-matrices $\mathcal{F}$, we define $\mathsf{forb}(m, r, \mathcal{F})$ as the maximum number of columns of an m-rowed, r-matrix A such that F is not a row-column permutation of A for all $F \in \mathcal{F}$. We investigate the asymptotic bounds of special families of forbidden configurations $\mathsf{Sym}(F)$, a symmetric family of matrices based on the $(0, 1)$-matrix F. The previously known lower bound constructions required F to have no constant row. We introduce a new lower bound construction that drops this requirement and is a better bound in certain cases. We will also investigate the effects of constant rows from the upper bound perspective, including upper bounds of block matrices, matrices consisting of entirely constant rows.

On the number of circuits and bases in matroids, of minimal codewords in a linear code and of support sets in exponential families

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We address the following questions and their relations:

What is the minimum and maximum number of circuits (minimal dependent sets) and bases in matroids of given size and rank, or in binary matroids?

What is the minimum value of number of minimal codewords for a code C of given length and dimension?

What is the minimum number of cycles in 3-connected cubic graph on n vertices, or in 3-connected graphs with p vertices and q edges?

What is the minimum number of support sets in exponential families?

Directed hypergraph problems

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We consider directed hypergraphs, which generalize directed graphs by allowing more than one vertex in the tail of a hyperedge (but only a single head vertex). There are different ways to define a path in a directed hypergraph. We discuss the version usually referred to as a 'hyperpath', which corresponds to natural notions in applications, for example, to the notion of a proof in some models of reasoning.

Domination game on uniform hypergraphs

Máté Vizer
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We introduce and study the domination game on hypergraphs. This is played on a hypergraph $\mathcal{H}$ by two players, namely Dominator and Staller, who alternately select vertices such that each selected vertex enlarges the set of vertices dominated so far. The game is over if all vertices of $\mathcal{H}$ are dominated. Dominator aims to finish the game as soon as possible, while Staller aims to delay the end of the game. If each player plays optimally and Dominator starts, the length of the game is the invariant *game domination number* denoted by $\gamma_g(\mathcal{H})$. This definition is the generalization of the domination game played on graphs and it is a special case of the transversal game on hypergraphs. After some basic general results, we establish an asymptotically tight (as k tends to infinity) upper bound on the game domination number of k-uniform hypergraphs. We also prove stronger results for small k's: e.g. we prove $\gamma_g(\mathcal{H}) \leq 5n/9$ if $\mathcal{H}$ is a 3-uniform hypergraph of order n and does not contain isolated vertices. This also implies the following new result for graphs: If G is an isolate-free graph on n vertices and each of its edges is contained in a triangle, then $\gamma_g(G) \leq 5n/9$.

Joint work with Cs. Bujtás, B. Patkós and Zs. Tuza.